UCB-PTH 04/06
LBNL-54674

Warped Supersymmetric Unification with Non-Unified Superparticle Spectrum

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Abstract

We present a new supersymmetric extension of the standard model. The model is constructed in warped space, with a unified bulk symmetry broken by boundary conditions on both the Planck and TeV branes. In the supersymmetric limit, the massless spectrum contains exotic colored particles along with the particle content of the minimal supersymmetric standard model (MSSM). Nevertheless, the model still reproduces the MSSM prediction for gauge coupling unification and does not suffer from a proton decay problem. The exotic states acquire masses from supersymmetry breaking, making the model completely viable, but there is still the possibility that these states will be detected at the LHC. The lightest of these states is most likely $A_5^{\rm XY}$, the fifth component of the gauge field associated with the broken unified symmetry. Because supersymmetry is broken on the $SU(5)$-violating TeV brane, the gaugino masses generated at the TeV scale are completely independent of one another. We explore some of the unusual features that the superparticle spectrum might have as a consequence.

1 Introduction

Because it stabilizes the weak scale and leads naturally to gauge coupling unification, weak-scale supersymmetry has attracted much interest. For many, the unification of couplings suggests that some new unified physics emerges at the scale at which the couplings meet [1]. But this scale is enormously large, and so our experimental probes of it are quite limited: we must rely on searches for extremely rare processes such as proton decay, or possibly on indirect hints, such as relations among the gaugino masses.

Models of unification in warped space, on the other hand, predict light particles whose quantum numbers are characteristic of an underlying enlarged symmetry. In the 5D description of these models, this enlarged symmetry is a grand unified gauge symmetry realized in the bulk, while in the dual 4D description, it is a global symmetry explicitly broken by the standard model gauge interactions. A complete, realistic model of warped supersymmetric unification was constructed in Ref. [2], building on an earlier suggestion [3], and on subsequent developments in our understanding of gauge coupling evolution in warped space [4 – 8] (a non-supersymmetric model has been discussed in [9]). As emphasized in [2], this framework leaves many of the most attractive features of conventional unification intact. For example, in the minimal supersymmetric standard model (MSSM), the assumption of unification yields an accurate prediction relating the values of the gauge couplings at low energies; in warped supersymmetric unification, a strong-coupling assumption leads to the same prediction at leading-log level. Yet physics at accessible energies could be quite different than in the conventional scenario. The model reveals its higher-dimensional nature at the TeV scale, through the appearance of Kaluza-Klein (KK) towers and an $N = 2$ supermultiplet structure.

In this paper we construct a model of warped unification in which the deviations from conventional expectations are perhaps even more striking than in [2]. Before supersymmetry breaking, the *massless* spectrum of the theory includes not only the states of the MSSM, but also exotic descendants of the enlarged symmetry. As in the model of [2], however, these extra particles do not spoil gauge coupling unification. From the 5D perspective, this setup arises when the grand unified gauge symmetry is broken by boundary conditions both on the Planck brane and on the TeV brane. From the 4D perspective, it arises when the approximate global symmetry is broken spontaneously by the strong dynamics associated with the conformal sector. The extra massless states make up the supermultiplets containing the pseudo-Goldstone bosons associated with this spontaneous breakdown.

We assume that the strong dynamics that break the global symmetry of the conformal sector also break supersymmetry, thereby giving masses to the particles in the pseudo-Goldstone multiplets. This link between the spontaneous breakdown of supersymmetry and the sponta-

neous breakdown of the enlarged symmetry is another important difference between this model and more conventional supersymmetric unification. Here there is no reason why the underlying unified symmetry should leave its imprint on the MSSM gaugino masses in any way — the bino might even be heavier than the gluino. From the 5D perspective, this is simply a consequence of the fact that the various supersymmetry-breaking gaugino masses on the TeV brane are completely independent, as the unified symmetry is not realized there. As a consequence, squark and slepton masses also do not obey the pattern characteristic to typical unified theories. This is in contrast to the situation in the model of [2], where the masses of all superparticles are characterized essentially by a single unified supersymmetry-breaking gaugino mass term [10].

The model we will study, then, is a supersymmetric extension of the standard model that incorporates gauge coupling unification, has a massless spectrum in the supersymmetric limit that differs substantially from that of the MSSM (the extra states do not even fill out full $SU(5)$ multiplets), and features a superpartner spectrum that generically looks nothing like those usually encountered in supersymmetric unification. We will see that such a model can be made fully realistic by introducing TeV-brane operators that transmit supersymmetry breaking to the pseudo-Goldstone multiplet. We will also calculate the spectrum of light states, and identify certain characteristic features of the superparticle spectrum that may be tested at future collider experiments.

The organization of the paper is as follows. In section 2 we construct our model. Supersymmetry breaking and its effects on various multiplets are discussed in section 3, and the spectrum is calculated. The phenomenology of the model is discussed in section 4. The model presented in this paper does not explicitly address the issues of charge quantization or quark-lepton unification. A model accommodating these features will be presented in a separate paper [11].

2 Model

In this section we construct our model. The construction closely follows that of Ref. [2]. The model is formulated in a 5D warped spacetime with the extra dimension compactified on an S^1/Z_2 orbifold: $0 \leq y \leq \pi R$, where y represents the coordinate of the extra dimension. The metric is given by

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{1}$$

where k is the AdS curvature, which is taken to be somewhat (typically a factor of a few) smaller than the 5D Planck scale M_5. The 4D Planck scale, $M_{\rm Pl}$, is given by $M_{\rm Pl}^2 \simeq M_5^3/k$ and we take $k \sim M_5 \sim M_{\rm Pl}$. We choose $kR \sim 10$ so that the TeV scale is naturally generated by the AdS warp factor: $k' \equiv k e^{-\pi k R} \sim$ TeV [12].[1]

[1]The quantity k' was denoted as T in Refs. [2, 10].

We consider a supersymmetric $SU(5)$ gauge theory on the above gravitational background. The bulk $SU(5)$ symmetry is broken by boundary conditions at both $y = 0$ and πR. Specifically, the 5D gauge multiplet can be decomposed into a 4D $N = 1$ vector superfield $V(A_\mu, \lambda)$ and a 4D $N = 1$ chiral superfield $\Sigma(\sigma + iA_5, \lambda')$, where both V and Σ are in the adjoint representation of $SU(5)$. The boundary conditions for these fields are given by

$$\begin{pmatrix} V \\ \Sigma \end{pmatrix}(x^\mu, -y) = \begin{pmatrix} PVP^{-1} \\ -P\Sigma P^{-1} \end{pmatrix}(x^\mu, y), \qquad \begin{pmatrix} V \\ \Sigma \end{pmatrix}(x^\mu, -y') = \begin{pmatrix} PVP^{-1} \\ -P\Sigma P^{-1} \end{pmatrix}(x^\mu, y'), \tag{2}$$

where $y' = y - \pi R$, and P is a 5×5 matrix acting on gauge space: $P = \mathrm{diag}(+,+,+,-,-)$. This reduces the gauge symmetry to $SU(3)_C \times SU(2)_L \times U(1)_Y$ (321) both at the $y = 0$ brane (Planck brane) and at the $y = \pi R$ brane (TeV brane). The gauge symmetry at low energies is $SU(3)_C \times SU(2)_L \times U(1)_Y$. The zero-mode sector contains not only the 321 component of V, but also the $SU(5)/(SU(3)_C \times SU(2)_L \times U(1)_Y)$ (XY) component of Σ. The typical mass scale for the KK towers is $k' \sim$ TeV, so that the lowest KK excitations of the standard model gauge fields and the lightest XY gauge bosons both have masses of order TeV.

The Higgs fields are introduced in the bulk as two hypermultiplets transforming as the fundamental representation of $SU(5)$. Using notation where a hypermultiplet is represented by two 4D $N = 1$ chiral superfields $\Phi(\phi, \psi)$ and $\Phi^c(\phi^c, \psi^c)$ with opposite gauge transformation properties, our two Higgs hypermultiplets can be written as $\{H, H^c\}$ and $\{\bar{H}, \bar{H}^c\}$, where H and $\bar{H}^c$ transform as $\mathbf{5}$ and $\bar{H}$ and H^c transform as $\mathbf{5}^*$ under $SU(5)$. The boundary conditions are given by

$$\begin{pmatrix} H \\ H^c \end{pmatrix}(x^\mu, -y) = \begin{pmatrix} -PH \\ PH^c \end{pmatrix}(x^\mu, y), \qquad \begin{pmatrix} H \\ H^c \end{pmatrix}(x^\mu, -y') = \begin{pmatrix} -PH \\ PH^c \end{pmatrix}(x^\mu, y'), \tag{3}$$

for $\{H, H^c\}$, and similarly for $\{\bar{H}, \bar{H}^c\}$. The zero modes consist of the $SU(2)_L$-doublet components of H and $\bar{H}$ and the $SU(3)_C$-triplet components of H^c and $\bar{H}^c$. A bulk hypermultiplet $\{\Phi, \Phi^c\}$ can generically have a mass term in the bulk, which is written as

$$S = \int d^4x \int_0^{\pi R} dy \left[e^{-3k|y|} \int d^2\theta \, c_\Phi k \Phi \Phi^c + \text{h.c.} \right], \tag{4}$$

in the basis where the kinetic term is given by $S_{\rm kin} = \int d^4x \int dy \, [e^{-2k|y|} \int d^4\theta (\Phi^\dagger \Phi + \Phi^c \Phi^{c\dagger}) + \{e^{-3k|y|} \int d^2\theta (\Phi^c \partial_y \Phi - \Phi \partial_y \Phi^c)/2 + \text{h.c.}\}]$ [13]. The parameter c_Φ controls the wavefunction profile of the zero mode. For $c_\Phi > 1/2$ ($< 1/2$) the wavefunction of a zero mode arising from Φ is localized to the Planck (TeV) brane; for $c_\Phi = 1/2$ it is conformally flat. If a zero mode arises from Φ^c, its wavefunction is localized to the TeV (Planck) brane for $c_\Phi > -1/2$ ($< -1/2$) and conformally flat for $c_\Phi = -1/2$. For the Higgs fields, we choose $c_H, c_{\bar{H}} \geq 1/2$ to preserve the

MSSM prediction for gauge coupling unification (see below).[2]

Matter fields are introduced on the Planck brane as a standard set of chiral superfields $Q(\mathbf{3},\mathbf{2})_{1/6}$, $U(\mathbf{3}^*,\mathbf{1})_{-2/3}$, $D(\mathbf{3}^*,\mathbf{1})_{1/3}$, $L(\mathbf{1},\mathbf{2})_{-1/2}$ and $E(\mathbf{1},\mathbf{1})_1$ for each generation. Here the numbers represent the transformation properties under $SU(3)_C \times SU(2)_L \times U(1)_Y$ with the $U(1)_Y$ charges normalized in the conventional way. With matter localized on the Planck brane, proton decay can be adequately suppressed despite the fact that XY gauge and colored Higgs fields have masses of order TeV. This is because the wavefunctions of the XY gauge and colored Higgs fields (and their KK excitations) are all strongly localized to the TeV brane. Yukawa couplings are introduced on the Planck brane:[3]

$$S = \int d^4x \int_0^{\pi R} dy\; 2\delta(y) \Big[\int d^2\theta \left(y_u QUH_D + y_d QD\bar{H}_D + y_e LE\bar{H}_D \right) + \text{h.c.} \Big], \tag{5}$$

where H_D and $\bar{H}_D$ are the doublet components of H and $\bar{H}$. The Yukawa couplings respect a $U(1)_R$ symmetry, under which the 4D superfields V, Σ, H and $\bar{H}$ are neutral, Q, U, D, L and E have unit charge, and H^c and $\bar{H}^c$ have charge $+2$, and we impose this symmetry on the theory. This $U(1)_R$ forbids dangerous dimension four and five proton decay operators together with a potentially large supersymmetric mass term for the Higgs fields [14] (the $U(1)_R$ symmetry is broken to its Z_2 subgroup through supersymmetry breaking but without reintroducing phenomenological problems). Small neutrino masses can be naturally generated through the conventional seesaw mechanism by introducing right-handed neutrino fields $N(\mathbf{1},\mathbf{1})_0$ with the Majorana mass terms and neutrino Yukawa couplings on the Planck brane:

$$S = \int d^4x \int_0^{\pi R} dy\; 2\delta(y) \Big[\int d^2\theta \left(\frac{M_N}{2} NN + y_\nu LNH_D \right) + \text{h.c.} \Big], \tag{6}$$

where N carries a $U(1)_R$ charge of $+1$.

We note that in our theory there is a priori no reason why the $U(1)_Y$ charges for the matter fields must obey the usual $SU(5)$ normalization (the one for which the Yukawa couplings of Eq. (5) are allowed). Thus our theory is not "grand unified" in the conventional sense. The correct normalization may be obtained by considering higher dimensional theories as in [15]; for instance by extending the $y = 0$ brane (the bulk) to a short 1-dimensional (thin 2-dimensional) object having $SU(4)_C \times SU(2)_L \times SU(2)_R$ ($SO(10)$) gauge symmetry. Alternatively, we could

[2]An alternative choice for the boundary conditions of the Higgs fields is given by Eq. (3) with an extra minus sign in the right-hand side of the second equation (i.e. flipping the TeV-brane boundary conditions). This also gives the MSSM prediction for gauge coupling unification for $c_H, c_{\bar{H}} \geq 1/2$, since the prediction does not depend on the physics at the TeV brane. Although these boundary conditions do not give a zero mode, four doublet states from H, H^c, $\bar{H}$ and $\bar{H}^c$ are relatively light [they are even exponentially lighter than k' for $c_H, c_{\bar{H}} > 1/2$ with the modes from H and $\bar{H}$ (H^c and $\bar{H}^c$) exponentially localized to the Planck (TeV) brane]. A realistic model is then obtained by giving a mass term of the form $\int d^2\theta H^c_D \bar{H}^c_D + \text{h.c.}$ on the TeV brane.

[3]Our convention for the delta function is $\int_0^\epsilon \delta(y) dy = 1/2$ for $\epsilon > 0$.

consider the conventional "grand unified theory" version of our theory, i.e. we could break $SU(5)$ by the brane-localized Higgs field at $y = 0$, although we then need some mechanism for doublet-triplet splitting on the brane. Finally, one could also consider a 5D theory with an enlarged bulk gauge group such as $SO(10)$, which will be discussed in Ref. [11].

Despite the presence of exotic low-energy states, the model still reproduces the successful prediction associated with MSSM gauge coupling unification. From the 5D viewpoint, there are three local operators that can contribute to the low-energy 4D gauge couplings:

$$S = -\frac{1}{4}\int d^4x \int_0^{\pi R} dy \left[\frac{1}{g_B^2} F_{\mu\nu}F^{\mu\nu} + 2\delta(y)\frac{1}{\tilde{g}_{0,a}^2} F^a{}_{\mu\nu}F^{a\mu\nu} + 2\delta(y-\pi R)\frac{1}{\tilde{g}_{\pi,a}^2} F^a{}_{\mu\nu}F^{a\mu\nu} \right], \qquad (7)$$

where g_B is the $SU(5)$-invariant 5D gauge coupling and the index a runs over $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ ($a = 3, 2, 1$, respectively). The structure of these terms is determined by the restricted 5D gauge symmetry, which reduces to 321 on the $y = 0$ and $y = \pi R$ branes but is $SU(5)$ in the bulk. At the fundamental scale $M_* \sim M_5$, the coefficients of these operators are incalculable parameters of the effective field theory, so that one might worry that the model may not give any prediction for the low-energy gauge couplings. This difficulty, however, can be avoided if we require that the entire theory is strongly coupled at the scale M_*. In this case the sizes of these coefficients are estimated as $1/g_B^2 \simeq M_*/16\pi^3$ and $1/\tilde{g}_{0,a}^2 \simeq 1/\tilde{g}_{\pi,a}^2 \simeq 1/16\pi^2$, and one finds that the low-energy prediction is insensitive to the parameters $\tilde{g}_{0,a}$ and $\tilde{g}_{\pi,a}$ evaluated at M_*. The prediction for the low-energy 4D gauge couplings, g_a, is then written in the form

$$\frac{1}{g_a^2(k')} \simeq (SU(5)\ \text{ symmetric}) + \frac{1}{8\pi^2}\Delta^a(k', k), \qquad (8)$$

where $\Delta^a(k', k)$ is the quantity whose non-universal part can be unambiguously computed in the effective theory. In the present model, this quantity is given at one-loop leading-log level by setting $(T_1, T_2, T_3)(V_{++}) = (0, 2, 3)$, $(T_1, T_2, T_3)(V_{--}) = (5, 3, 2)$, $(T_1, T_2, T_3)(V_{+-}) = (T_1, T_2, T_3)(V_{-+}) = (0, 0, 0)$ and $(T_1, T_2, T_3)(\Phi_{++}) = (3/10, 1/2, 0)$, $(T_1, T_2, T_3)(\Phi_{--}) = (1/5, 0, 1/2)$, $(T_1, T_2, T_3)(\Phi_{+-}) = (T_1, T_2, T_3)(\Phi_{-+}) = (0, 0, 0)$, $c_{++} = c_{--} \geq 1/2$ for $\Phi = H, \bar{H}$ in Eqs. (9) and (10) of Ref. [2], respectively. Adding everything together, we obtain

$$\begin{pmatrix} \Delta^1 \\ \Delta^2 \\ \Delta^3 \end{pmatrix} (k', k) \simeq \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} \ln\left(\frac{k}{k'}\right), \qquad (9)$$

where we have absorbed a possible $SU(5)$ symmetric piece into the first term of Eq. (8). This is exactly the relation obtained in conventional 4D supersymmetric unification with the parameter k identified with the unification scale. This result can be understood more intuitively by noticing that the gauge couplings above the TeV scale can be defined as the coefficients of the gauge two-point correlators whose end points are both on the Planck brane [5]. We then find that the light

extra states do not contribute to the large logarithm in Eq. (9) because they are all strongly localized to the TeV brane and the Planck-brane gauge correlators do not probe the region near the TeV brane at energies higher than k'. In a suitable renormalization scheme, the large logarithmic contribution can be absorbed in the couplings on the Planck brane. In this case the brane couplings renormalized at the scale $\mu \sim k'$ are given by

$$\frac{1}{\tilde{g}_{0,a}^2(k')} \simeq \frac{1}{8\pi^2}\Delta^a(k',k), \qquad \frac{1}{\tilde{g}_{\pi,a}^2(k')} = O\Big(\frac{1}{16\pi^2}\Big), \tag{10}$$

where $\Delta^a(k',k)$ are given by Eq. (9) and the scales are measured in terms of the 4D metric $\eta_{\mu\nu}$ [10]. We can thus safely neglect $1/\tilde{g}_{\pi,a}^2$ in any formulae given below (and we will) but not necessarily $1/\tilde{g}_{0,a}^2$.

Let us now work out the spectrum of the model in more detail, following the procedure of Ref. [10]. We first consider the gauge sector. The zero modes consist of the 321 component of V, V^{321}, and the XY component of Σ, $\Sigma^{\rm XY}$, which transform as $(\mathbf{8},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{1},\mathbf{1})_0$ and $(\mathbf{3},\mathbf{2})_{-5/6} + (\mathbf{3}^*,\mathbf{2})_{5/6}$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$, respectively. The KK excitations for the 321 fields consist of V^{321} and Σ^{321} at each KK level, whose masses m_n are determined by the equation

$$\frac{J_0\left(\frac{m_n}{k}\right) + \frac{g_B^2}{\tilde{g}_{0,a}^2} m_n J_1\left(\frac{m_n}{k}\right)}{Y_0\left(\frac{m_n}{k}\right) + \frac{g_B^2}{\tilde{g}_{0,a}^2} m_n Y_1\left(\frac{m_n}{k}\right)} = \frac{J_0\left(\frac{m_n}{k'}\right)}{Y_0\left(\frac{m_n}{k'}\right)}, \tag{11}$$

where $J_n(x)$ and $Y_n(x)$ are the Bessel functions of order n, and $\tilde{g}_{0,a}^2$ $(a=1,2,3)$ are given by Eq. (10).[4] For the XY fields, each KK excited level consists of $V^{\rm XY}$ and $\Sigma^{\rm XY}$ with the masses given by

$$\frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} = \frac{J_1\left(\frac{m_n}{k'}\right)}{Y_1\left(\frac{m_n}{k'}\right)}. \tag{12}$$

Therefore, for $k' \ll k$, the spectrum in the gauge sector is given by

$$\left\{ \begin{array}{ll} V^{321}: & m_0 = 0, \\ \{V^{321}, \Sigma^{321}\}: & m_n \simeq (n-\frac{1}{4})\pi k', \end{array} \right. \qquad \left\{ \begin{array}{ll} \Sigma^{\rm XY}: & m_0 = 0, \\ \{V^{\rm XY}, \Sigma^{\rm XY}\}: & m_n \simeq (n+\frac{1}{4})\pi k', \end{array} \right. \tag{13}$$

where $n = 1, 2, \cdots$. This spectrum is depicted for the lowest-lying modes in Fig. 1a. The resulting spectrum is quite different from that of the model discussed in [2, 10]. The massless sector contains the XY states $\Sigma^{\rm XY}$ as well as the MSSM gauge fields V^{321}, and the KK excited states are not $SU(5)$ symmetric even approximately. As we will see in the next section, the unwanted zero modes from $\Sigma^{\rm XY}$ obtain masses once supersymmetry is broken.

[4]The effects of brane-localized kinetic terms on the spectrum of the gauge boson KK towers were studied in [16].

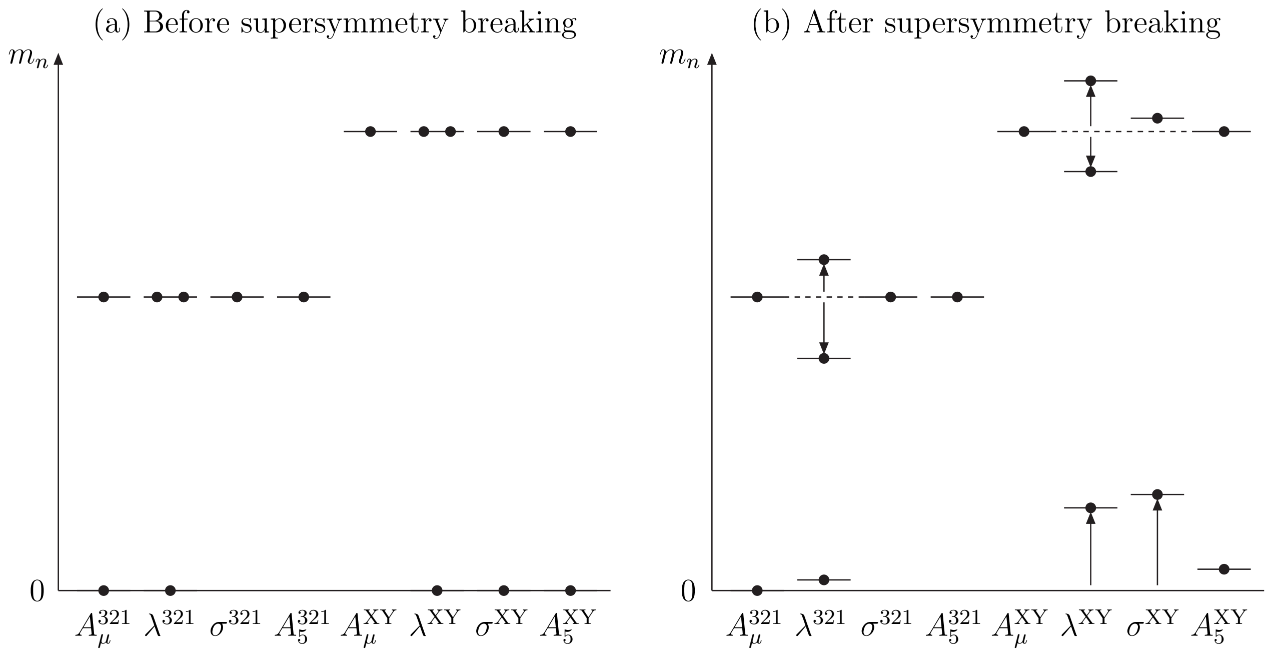

Figure 1: Schematic depiction for the lowest-lying masses for the 321 gauge multiplet (A_μ^{321}, λ^{321}, σ^{321}, A_5^{321}) and the XY gauge multiplet ($A_\mu^{\rm XY}$, $\lambda^{\rm XY}$, $\sigma^{\rm XY}$, $A_5^{\rm XY}$) (a) before and (b) after supersymmetry breaking. Each bullet for λ^{321} and $\lambda^{\rm XY}$ represents a Majorana and Dirac degree of freedom, respectively. Arrows indicate displacements of the mass levels relative to their supersymmetric positions, represented by dotted lines.

For the Higgs fields, the massless sector consists of two doublets H_D and $\bar{H}_D$ ($\subset H$ and $\bar{H}$) and two triplets H_T^c and $\bar{H}_T^c$ ($\subset H^c$ and $\bar{H}^c$). The excited states for the doublets and triplets contain $\{H_D, \bar{H}_D, H_D^c, \bar{H}_D^c\}$ and $\{H_T, \bar{H}_T, H_T^c, \bar{H}_T^c\}$, respectively. For $\{H, H^c\}$, the KK masses are given by

$$\frac{J_{c_H \mp 1/2}\left(\frac{m_n}{k}\right)}{Y_{c_H \mp 1/2}\left(\frac{m_n}{k}\right)} = \frac{J_{c_H \mp 1/2}\left(\frac{m_n}{k'}\right)}{Y_{c_H \mp 1/2}\left(\frac{m_n}{k'}\right)}, \tag{14}$$

where $\mp$ takes $-$ and $+$ for the doublets and triplets, respectively. Similarly, the masses for $\{\bar{H}, \bar{H}^c\}$ are given by Eq. (14) with c_H replaced by $c_{\bar{H}}$. Here, we have neglected possible Planck-brane localized kinetic terms for simplicity. The Higgs spectrum is thus given by

$$\left\{ \begin{array}{ll} \{H_D, \bar{H}_D\}: & m_0 = 0, \\ \{H_D, H_D^c\}: & m_n \simeq (n + \frac{c_H}{2} - \frac{1}{2})\pi k', \\ \{\bar{H}_D, \bar{H}_D^c\}: & m_n \simeq (n + \frac{c_{\bar{H}}}{2} - \frac{1}{2})\pi k', \end{array} \right. \qquad \left\{ \begin{array}{ll} \{H_T^c, \bar{H}_T^c\}: & m_0 = 0, \\ \{H_T, H_T^c\}: & m_n \simeq (n + \frac{c_H}{2})\pi k', \\ \{\bar{H}_T, \bar{H}_T^c\}: & m_n \simeq (n + \frac{c_{\bar{H}}}{2})\pi k', \end{array} \right. \tag{15}$$

where $n = 1, 2, \cdots$. For $c_H = c_{\bar{H}} = 1/2$ the Higgs spectrum is identical to the gauge spectrum with the doublet and triplet components corresponding to the 321 and XY components, respectively (up to small difference arising from the Planck-brane localized operators). For $c_H, c_{\bar{H}} > 1/2$, the Higgs KK towers are heavier than the gauge towers. After supersymmetry breaking, the $\{H, H^c\}$ and $\{\bar{H}, \bar{H}^c\}$ states are mixed and the spectrum is distorted through the supersymmetric and supersymmetry-breaking mass terms generated on the TeV brane (or on the Planck brane depending on the details of the Higgs sector). The zero modes in Eq. (15) obtain masses through these operators.

It is useful to understand the structure of the model described above in terms of the 4D dual description of the theory. Through the AdS/CFT duality [17], applied to a truncated space [18], we can relate our 5D model to a purely 4D theory. The 4D theory is defined at the ultraviolet (UV) cutoff scale of order $k \sim M_{\rm Pl}$ and contains a gauge interaction with the group G, whose coupling $\tilde{g}$ evolves very slowly over a wide energy interval below k. Denoting the size of the group G to be N, the correspondence is given by $\tilde{g}^2 N/16\pi^2 \approx M_*/\pi k$ and $N \approx 16\pi^2/g_B^2 k$ (so $\tilde{g} \simeq 4\pi$ and $N \gtrsim 1$ here). The bulk gauge symmetry and the Planck-brane boundary conditions in the 5D theory imply that the G gauge sector possesses a global $SU(5)$ symmetry whose $SU(3)_C \times SU(2)_L \times U(1)_Y$ subgroup is explicitly gauged. The fields singlet under G correspond to the modes localized to the Planck brane, i.e. the MSSM quark, lepton and Higgs fields. The theory below k, therefore, appears as a supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y \times G$ gauge theory with the quarks, leptons and two Higgs doublets transforming under $SU(3)_C \times SU(2)_L \times U(1)_Y$. In this 4D picture, a prediction relating the gauge couplings at low energies arises because the gauge couplings of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ become strong at the UV scale k due to the asymptotically non-free contribution from G [2]. Since the G contribution is $SU(5)$

symmetric, the prediction is determined by the contributions from the matter, Higgs and 321 gauge multiplets, reproducing the successful MSSM prediction.

At the TeV scale the gauge interaction of G becomes strong and exhibits non-trivial infrared (IR) phenomena, corresponding to the presence of the TeV brane in the 5D picture. The boundary conditions of Eq. (2) then imply that the global $SU(5)$ symmetry of the G sector is spontaneously broken to $SU(3)_C \times SU(2)_L \times U(1)_Y$ by the IR dynamics of G. The IR dynamics of G also produces resonances of masses of order TeV. These resonances have a tower structure and correspond to the KK towers in the 5D picture. Because the global $SU(5)$ of G is dynamically broken, the spectrum of the towers does not respect $SU(5)$, as shown in Eqs. (13, 15). Moreover, the dynamical breaking of $SU(5)$ to 321 produces Goldstone bosons, which correspond to the XY components of A_5 in an appropriate basis. This implies that in the absence of supersymmetry breaking the entire 4D supermultiplet containing $A_5^{\rm XY}$ (i.e. $\Sigma^{\rm XY}$) must be massless, in agreement with Eq. (13). In fact, because of the partial gauging of $SU(5)$, the global $SU(5)$ of G is explicitly broken by the 321 gauge interactions and the $A_5^{\rm XY}$ is only a pseudo-Goldstone boson. This, however, does not give any mass for $\Sigma^{\rm XY}$ as long as supersymmetry is unbroken.

It is now relatively clear what happens when supersymmetry is broken. Supersymmetry breaking can be caused by the dynamics of G, which is the situation we will consider in the next section. It then gives masses of order TeV to $\sigma^{\rm XY}$ and $\lambda'^{\rm XY}$, that is, to all of $\Sigma^{\rm XY}$ except for $A_5^{\rm XY}$. The breaking of supersymmetry is accompanied by that of $U(1)_R$, and the MSSM gauginos and Higgs multiplets also obtain masses. Squarks and sleptons are massless at the leading order because they do not directly interact with the G sector. Their masses, however, are generated at one loop through the 321 gauge interactions. Since these masses are flavor universal, the supersymmetric flavor problem is absent. The mass of $A_5^{\rm XY}$ is also generated through the 321 gauge loop, since it picks up the explicit breaking of the global $SU(5)$. The only massless fields remaining after supersymmetry breaking are the standard-model quarks, leptons and gauge fields. Both the MSSM superparticles and exotic grand-unified-theoretic (GUT) states such as $\Sigma^{\rm XY}$ obtain masses, although the exotic states are generically expected to be heavier because they are composite states of G and thus more strongly coupled to supersymmetry breaking caused by G.

In the next section we discuss supersymmetry breaking in more detail using the 5D picture. It will be shown how the dynamical supersymmetry breaking scenario outlined above is realized in 5D. The spectrum of the lowest-lying modes and its various interesting features will also be worked out.

3 Supersymmetry Breaking

As we have seen in the previous section, the massless sector of our model before supersymmetry breaking contains a chiral superfield with the quantum numbers of the XY gauge bosons, $\Sigma^{\rm XY}$, and a vector-like pair of color-triplet Higgs fields, H_T^c and $\bar{H}_T^c$, in addition to the usual MSSM fields. After supersymmetry is broken, all these fields must obtain masses, along with the MSSM superparticles. In this section we discuss how these masses arise and what the spectrum of the extra states and the MSSM superparticles looks like. We also present a complete set of formulae giving the masses for the light states as well as the KK towers.

We consider the case where supersymmetry is broken on the TeV brane. In terms of the 4D picture, this corresponds to a situation in which supersymmetry is broken by the dynamics of G at the TeV scale. Assuming that $U(1)_R$ is also broken by these dynamics, we can parameterize their effects by a supersymmetry (and $U(1)_R$-symmetry) breaking vacuum expectation value. Specifically, we introduce a singlet chiral field Z on the TeV brane together with the superpotential [19]:

$$S = \int d^4x \int_0^{\pi R} dy\ 2\delta(y-\pi R)\Big[e^{-2\pi kR}\int d^4\theta Z^\dagger Z + \Big\{e^{-3\pi kR}\int d^2\theta \Lambda^2 Z + {\rm h.c.}\Big\}\Big], \qquad (16)$$

which produces the desired vacuum expectation value $\langle Z\rangle = -e^{-\pi kR}\Lambda^{*2}\theta^2$, where Λ is a mass parameter of order $M_* \sim M_5$. Here, we have assumed that higher powers in Z are absent in the superpotential, and that the flat direction of Z is stabilized by higher order terms in the Kähler potential (for a dynamical model achieving this, see e.g. [20]). This will give TeV-scale masses for various extra states and the MSSM superparticles, which we will discuss in turn. As was discussed in [2], this breaking does not disturb the prediction relating the low-energy gauge couplings.

Let us begin by the gauge sector of the model. The gauge kinetic terms are given by [13][5]

$$\begin{aligned} S \;=\; & \int d^4x \int_0^{\pi R} dy \Bigg[\Big\{\frac{1}{2g_B^2}\int d^2\theta\, {\rm Tr}[\mathcal{W}^\alpha \mathcal{W}_\alpha] + {\rm h.c.}\Big\} + \frac{e^{-2k|y|}}{2g_B^2}\int d^4\theta\, {\rm Tr}[\mathcal{A}^2] \\ & + 2\delta(y)\sum_{a=1,2,3}\Big\{\frac{1}{2\tilde{g}_{0,a}^2}\int d^2\theta\, {\rm Tr}[\mathcal{W}_a^\alpha \mathcal{W}_{a\alpha}] + {\rm h.c.}\Big\}\Bigg], \end{aligned} \qquad (17)$$

where

$$\mathcal{A} \equiv e^{-V}(\partial_y e^V) + (\partial_y e^V)\, e^{-V} - \sqrt{2}\, e^V \Sigma\, e^{-V} - \sqrt{2}\, e^{-V}\Sigma^\dagger e^V, \qquad (18)$$

$\mathcal{W}_\alpha \equiv -(1/8)\bar{\mathcal{D}}^2(e^{-2V}\mathcal{D}_\alpha e^{2V})$ is the $SU(5)$ field-strength superfield, and $\mathcal{W}_{a\alpha}$ with $a = 1,2,3$ are the field-strength superfields for the $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ subgroups, respectively

[5] In this paper we give all the expressions in the Wess-Zumino gauge.

($\mathcal{W}_{a\alpha} \subset \mathcal{W}_\alpha$). The MSSM gaugino masses then arise from the following operators:

$$S = \int d^4x \int_0^{\pi R} dy\, 2\delta(y-\pi R) \sum_{a=1,2,3} \Big[-\int d^2\theta\, \frac{\zeta_a}{M_*} Z\, \mathrm{Tr}[\mathcal{W}_a^\alpha \mathcal{W}_{a\alpha}] + \mathrm{h.c.} \Big], \tag{19}$$

where ζ_a are dimensionless parameters. Note that the operators in Eq. (19) do not respect the $SU(5)$ symmetry, as $SU(5)$ is broken to 321 on the TeV brane by boundary conditions, and so these operators generate non-universal gaugino masses at the TeV scale.[6] Specifically, the masses of the 321 gauginos (and their KK towers) are given as the solution to the following equation:

$$\frac{J_0\left(\frac{m_n}{k}\right) + \frac{g_B^2}{\tilde{g}_{0,a}^2} m_n J_1\left(\frac{m_n}{k}\right)}{Y_0\left(\frac{m_n}{k}\right) + \frac{g_B^2}{\tilde{g}_{0,a}^2} m_n Y_1\left(\frac{m_n}{k}\right)} = \frac{J_0\left(\frac{m_n}{k'}\right) + g_B^2 M_{\lambda,a} J_1\left(\frac{m_n}{k'}\right)}{Y_0\left(\frac{m_n}{k'}\right) + g_B^2 M_{\lambda,a} Y_1\left(\frac{m_n}{k'}\right)}, \tag{20}$$

where $M_{\lambda,a} \equiv \zeta_a \Lambda^{*2}/M_*$ [10] (see also [21]). Note that here we have to look for the solutions with $m_n < 0$ as well as those with $m_n > 0$ to obtain all the masses (the physical masses are given by $|m_n|$). The non-universal nature of the gaugino masses becomes important when we discuss squark and slepton masses and the phenomenology of the model.

The masses for fields in $\Sigma^{\rm XY}$ are generated through the operator

$$S = \int d^4x \int_0^{\pi R} dy\, 2\delta(y-\pi R) \Big[e^{-2\pi kR} \int d^4\theta\, \frac{\eta}{2M_*} Z^\dagger\, \mathrm{Tr}[\mathcal{P}[\mathcal{A}]\mathcal{P}[\mathcal{A}]] + \mathrm{h.c.} \Big], \tag{21}$$

where the trace is over the $SU(5)$ space and $\mathcal{P}[\mathcal{X}]$ is a projection operator: with $\mathcal{X}$ an adjoint of $SU(5)$, $\mathcal{P}[\mathcal{X}]$ extracts the $(\mathbf{3},\mathbf{2})_{-5/6} + (\mathbf{3}^*,\mathbf{2})_{5/6}$ component of $\mathcal{X}$ under the decomposition to 321. The coefficient η is a dimensionless parameter. Note that $\mathcal{P}[\mathcal{A}]$ is even under the parity $y' \to -y'$ so that the above operator is non-vanishing. This operator gives masses to the fermion component $\lambda'^{\rm XY}$ and to the real-scalar component $\sigma^{\rm XY}$ contained in $\Sigma^{\rm XY}$. The mass of $\lambda'^{\rm XY}$, and in fact the masses of the entire XY gaugino KK tower consisting of $\lambda^{\rm XY}$ and $\lambda'^{\rm XY}$, are given by the equation

$$\frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} = \frac{J_1\left(\frac{m_n}{k'}\right) - g_B^2 M_{\lambda,X} J_0\left(\frac{m_n}{k'}\right)}{Y_1\left(\frac{m_n}{k'}\right) - g_B^2 M_{\lambda,X} Y_0\left(\frac{m_n}{k'}\right)}, \tag{22}$$

where $M_{\lambda,X} \equiv \eta\Lambda^2/M_*$. Here, we have not included a possible kinetic term for $\Sigma^{\rm XY}$ on the Planck brane, but this term barely affects the spectrum since the $\lambda'^{\rm XY}$'s are strongly localized to the TeV brane. For $\sigma^{\rm XY}$, we find a term proportional to $\delta(y-\pi R)^2$ in its equation of motion after integrating out the auxiliary field in $\Sigma^{\rm XY}$. This singular term, however, is canceled by appropriately choosing the coefficient of the operator

$$S = \int d^4x \int_0^{\pi R} dy\, 2\delta(y-\pi R) \Big[-e^{-2\pi kR} \int d^4\theta\, \frac{\rho}{4M_*^2} Z^\dagger Z\, \mathrm{Tr}[\mathcal{P}[\mathcal{A}]\mathcal{P}[\mathcal{A}]] \Big], \tag{23}$$

[6] The mechanism of generating non-universal gaugino masses on an $SU(5)$-violating brane was considered in flat-space models in [14].

which also gives a mass for $\sigma^{\rm XY}$. The consistency of the effective theory then requires ρ to take the form $\rho = -8 g_B^2 |\eta|^2 \delta(0) + \rho'$, where ρ' is a dimensionless parameter. Here the first term in ρ is chosen such that it cancels the singular term arising from the operator in Eq. (21). In fact, this singular term in ρ is simply a counterterm chosen to absorb divergences order by order in perturbation theory, although it appears already at tree level. The masses of $\sigma^{\rm XY}$ and their KK towers are then given by

$$\frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} = \frac{J_1\left(\frac{m_n}{k'}\right) - \frac{g_B^2 M_{\sigma,X}^2 k'}{m_n k} J_0\left(\frac{m_n}{k'}\right)}{Y_1\left(\frac{m_n}{k'}\right) - \frac{g_B^2 M_{\sigma,X}^2 k'}{m_n k} Y_0\left(\frac{m_n}{k'}\right)}, \tag{24}$$

where $M_{\sigma,X}^2 \equiv \rho' |\Lambda|^4 / M_*^2$. The mass of $A_5^{\rm XY}$ does not directly arise from the operators in Eqs. (21, 23), as it is protected by the 5D gauge invariance (in the 4D picture $A_5^{\rm XY}$ is a pseudo-Goldstone boson of $SU(5) \rightarrow 321$ caused by the dynamics of G). However, it arises through loop effects as we will discuss later.

The Higgs multiplets, H_D and $\bar{H}_D$, obtain masses from the terms

$$\begin{aligned} S &= \int d^4x \int_0^{\pi R} dy\, 2\delta(y-\pi R)\, e^{-2\pi k R} \int d^4\theta \left[\frac{\zeta_D}{M_*^2} Z^\dagger H_D \bar{H}_D + \frac{\zeta_D^*}{M_*^2} Z H_D^\dagger \bar{H}_D^\dagger \right. \\ & \left. - \frac{\rho_D}{M_*^3} Z^\dagger Z H_D \bar{H}_D - \frac{\rho_D^*}{M_*^3} Z^\dagger Z H_D^\dagger \bar{H}_D^\dagger - \frac{\eta_{H_D}}{M_*^3} Z^\dagger Z H_D^\dagger H_D - \frac{\eta_{\bar{H}_D}}{M_*^3} Z^\dagger Z \bar{H}_D^\dagger \bar{H}_D \right], \end{aligned} \tag{25}$$

where ζ_D, ρ_D, η_{H_D} and $\eta_{\bar{H}_D}$ are dimensionless parameters. Similarly, the H_T^c and $\bar{H}_T^c$ fields obtain masses from terms as in Eq. (25) with the fields $\{H_D, \bar{H}_D\}$ and couplings $\{\zeta_D, \rho_D, \eta_{H_D}, \eta_{\bar{H}_D}\}$ replaced by $\{H_T^c, \bar{H}_T^c\}$ and $\{\zeta_T, \rho_T, \eta_{H_T}, \eta_{\bar{H}_T}\}$, respectively. The terms in the first line of Eq. (25) (and the corresponding ones for the Higgs triplets) give supersymmetric mass terms for H_D and $\bar{H}_D$ (and for H_T^c and $\bar{H}_T^c$). The masses for the fermionic components of the Higgs multiplets are then determined by the following equation:

$$I_{H,R}^{(1)} I_{\bar{H},R}^{(1)} - |\tilde{\zeta}_R|^2 I_{H,R}^{(2)} I_{\bar{H},R}^{(2)} = 0, \tag{26}$$

where $\tilde{\zeta}_R \equiv -\zeta_R \Lambda^{*2} / M_*^2$ and the index $R = D, T$ represents the doublet and triplet components. The functions $I_{\Phi,R}^{(1)}$ and $I_{\Phi,R}^{(2)}$ are defined by

$$I_{\Phi,R}^{(1)} = J_{c_\Phi \mp \frac{1}{2}}\Big(\frac{m_n}{k'}\Big) - \frac{J_{c_\Phi \mp \frac{1}{2}}(\frac{m_n}{k})}{Y_{c_\Phi \mp \frac{1}{2}}(\frac{m_n}{k})} Y_{c_\Phi \mp \frac{1}{2}}\Big(\frac{m_n}{k'}\Big), \tag{27}$$

$$I_{\Phi,R}^{(2)} = J_{c_\Phi \pm \frac{1}{2}}\Big(\frac{m_n}{k'}\Big) - \frac{J_{c_\Phi \mp \frac{1}{2}}(\frac{m_n}{k})}{Y_{c_\Phi \mp \frac{1}{2}}(\frac{m_n}{k})} Y_{c_\Phi \pm \frac{1}{2}}\Big(\frac{m_n}{k'}\Big), \tag{28}$$

where $\Phi = H, \bar{H}$, and $\mp$ ($\pm$) takes $-$ and $+$ ($+$ and $-$) for the doublets, $R = D$, and triplets, $R = T$, respectively. The scalar components receive contributions also from the terms in the

second line in Eq. (25) (and the corresponding ones for the triplets). Their masses are given by

$$\Big(I^{(1)}_{H,R}I^{(1)}_{\bar{H},R} - |\tilde{\zeta}_R|^2 I^{(2)}_{H,R}I^{(2)}_{\bar{H},R} + \tilde{\eta}_{H_R}\frac{M'_*}{m_n}I^{(1)}_{\bar{H},R}I^{(2)}_{H,R}\Big)\Big(I^{(1)}_{H,R}I^{(1)}_{\bar{H},R} - |\tilde{\zeta}_R|^2 I^{(2)}_{H,R}I^{(2)}_{\bar{H},R} + \tilde{\eta}_{\bar{H}_R}\frac{M'_*}{m_n}I^{(1)}_{H,R}I^{(2)}_{\bar{H},R}\Big)$$
$$-|\tilde{\rho}_R|^2\frac{M'^2_*}{m_n^2}I^{(1)}_{H,R}I^{(1)}_{\bar{H},R}I^{(2)}_{H,R}I^{(2)}_{\bar{H},R} = 0, \tag{29}$$

where $\tilde{\rho}_R \equiv \rho_R|\Lambda|^4/M_*^4$ and $\tilde{\eta}_{\Phi_R} \equiv \eta_{\Phi_R}|\Lambda|^4/M_*^4$. This equation reduces to Eq. (26) for $\tilde{\rho}_R = \tilde{\eta}_{\Phi_R} = 0$. In Eqs. (26, 29) we have not included possible Planck-brane localized kinetic terms for H_D and $\bar{H}_D$, $S = \int d^4x \int dy\, 2\delta(y) \int d^4\theta [(z_H/M_*)H_D^\dagger H_D + (z_{\bar{H}}/M_*)\bar{H}_D^\dagger \bar{H}_D]$. Their effects are included by replacing $J_{c_\Phi-1/2}(m_n/k)$ and $Y_{c_\Phi-1/2}(m_n/k)$ in Eqs. (27, 28) with $J_{c_\Phi-1/2}(m_n/k) + (z_\Phi m_n/M_*)J_{c_\Phi+1/2}(m_n/k)$ and $Y_{c_\Phi-1/2}(m_n/k)+(z_\Phi m_n/M_*)Y_{c_\Phi+1/2}(m_n/k)$, respectively. The corresponding terms for H_T^c and $\bar{H}_T^c$ do not affect the spectrum because the triplet fields are localized to the TeV brane.

It is useful to study the relative sizes of the masses obtained above. For small supersymmetry breaking $\Lambda \ll M_*$, we can expand Eqs. (20, 22, 24, 26, 29) in powers of Λ/M_*. The masses of the lowest-lying modes in the gauge sector, λ^{321}, $\lambda'^{\rm XY}$ and $\sigma^{\rm XY}$, are then given by

$$m_{\lambda^{321}_a} = g_a^2 M'_{\lambda,a}, \tag{30}$$

$$m_{\lambda'^{\rm XY}} = 2g_B^2 k M'_{\lambda,X}, \tag{31}$$

$$m^2_{\sigma^{\rm XY}} = 2g_B^2 k M'^2_{\sigma,X}, \tag{32}$$

respectively, where $a = 1, 2, 3$ represents the 321 gauge group. Here, $M'_{\lambda,a} \equiv M_{\lambda,a}e^{-\pi kR}$, $M'_{\lambda,X} \equiv M_{\lambda,X}e^{-\pi kR}$ and $M'_{\sigma,X} \equiv M_{\sigma,X}e^{-\pi kR}$ are parameters of order TeV, and $g_a \equiv (\pi R/g_B^2 + 1/\tilde{g}_{0,a}^2)^{-1/2}$ are the 4D gauge couplings. Since $g_a = O(1)$ and $g_B^2 k = O(\pi kR)$ in the present theory, we find that the XY states, $\lambda'^{\rm XY}$ and $\sigma^{\rm XY}$, are expected to be heavier than the 321 gauginos, λ_a^{321}. For instance, in the case of $M_{\lambda,a} \simeq M_{\lambda,X} \simeq M_{\sigma,X}/4\pi$, as suggested by naive dimensional analysis, the ratios of the masses are given by $m_{\lambda^{321}_a} : m_{\lambda'^{\rm XY}} : m_{\sigma^{\rm XY}} \simeq 1 : \pi kR : \pi kR$ (here we have regarded $\sqrt{\pi kR}$ to be $O(4\pi)$ in this estimate). This can be understood in the 5D picture by the fact that the wavefunctions of the XY states are strongly localized to the TeV brane, where supersymmetry breaking occurs, while those of the 321 gauginos are nearly conformally flat. In the normalization where conformally flat modes have flat wavefunctions, the wavefunctions for λ^{321}, $\lambda'^{\rm XY}$ and $\sigma^{\rm XY}$ are roughly proportional to 1, $e^{k|y|}$ and $e^{k|y|}$, respectively. In the 4D dual picture the heaviness of the XY states arises because these states are composite states of the G sector so that they interact more strongly with supersymmetry breaking caused by the dynamics of G.

Similarly in the Higgs sector we can obtain the expressions for the masses in the small supersymmetry breaking limit. For the doublet Higgs fields, the masses are expressed by the

conventional set of parameters: the supersymmetric mass μ, the holomorphic supersymmetry-breaking scalar mass μB, and non-holomorphic supersymmetry-breaking squared masses m_Φ^2 ($\Phi = H, \bar{H}$). Defining

$$\delta_\Phi = \frac{1 - e^{-(2c_\Phi - 1)\pi k R}}{2c_\Phi - 1} + \frac{z_\Phi k}{M_*}, \tag{33}$$

these parameters are given by

$$\mu = \delta_H^{-1/2} \delta_{\bar{H}}^{-1/2} e^{-(c_H - \frac{1}{2})\pi k R} e^{-(c_{\bar{H}} - \frac{1}{2})\pi k R} \tilde{\zeta}_D \, k', \tag{34}$$

$$\mu B = \delta_H^{-1/2} \delta_{\bar{H}}^{-1/2} e^{-(c_H - \frac{1}{2})\pi k R} e^{-(c_{\bar{H}} - \frac{1}{2})\pi k R} \tilde{\rho}_D \, k' M'_*, \tag{35}$$

$$m_\Phi^2 = \delta_\Phi^{-1} e^{-2(c_\Phi - \frac{1}{2})\pi k R} \tilde{\eta}_{\Phi_D} \, k' M'_*. \tag{36}$$

The masses for the triplet Higgs states are also specified by their supersymmetric mass, holomorphic supersymmetry-breaking scalar mass and non-holomorphic supersymmetry-breaking squared masses, which are denoted as μ_T, $\mu_T B_T$ and $m_{\Phi_T}^2$, respectively. Defining

$$\delta_{\Phi_T} = \frac{1 - e^{-(2c_\Phi + 1)\pi k R}}{2c_\Phi + 1}, \tag{37}$$

they are given by

$$\mu_T = \delta_{H_T}^{-1/2} \delta_{\bar{H}_T}^{-1/2} \tilde{\zeta}_T \, k', \tag{38}$$

$$\mu_T B_T = \delta_{H_T}^{-1/2} \delta_{\bar{H}_T}^{-1/2} \tilde{\rho}_T \, k' M'_*, \tag{39}$$

$$m_{\Phi_T}^2 = \delta_{\Phi_T}^{-1} \tilde{\eta}_{\Phi_T} \, k' M'_*. \tag{40}$$

Comparing Eqs. (34 – 36) and Eqs. (38 – 40), we find that the masses of the triplet states tend to be larger than those of the doublet states. For instance, if all the dimensionless parameters are of the same order as expected in naive dimensional analysis, the ratios of the masses are given by $\mu/\mu_T \simeq \mu B/\mu_T B_T \simeq m_\Phi^2/m_{\Phi_T}^2 = O(1/\pi k R)$ for $c_H \simeq c_{\bar{H}} \simeq 1/2$, and $\mu/\mu_T \simeq \mu B/\mu_T B_T = O(e^{-(c_H + c_{\bar{H}} - 1)\pi k R})$ and $m_\Phi^2/m_{\Phi_T}^2 = O(e^{-(2c_\Phi - 1)\pi k R})$ for $c_H, c_{\bar{H}} > 1/2$. This is again because the wavefunctions for the triplet states are localized to the TeV brane while those of the doublet states are not. In the normalization where conformally flat modes have flat wavefunctions, the wavefunctions for the H_D, $\bar{H}_D$, H_T^c and $\bar{H}_T^c$ states are roughly proportional to $e^{-(c_H - 1/2)k|y|}$, $e^{-(c_{\bar{H}} - 1/2)k|y|}$, $e^{(c_H + 1/2)k|y|}$ and $e^{(c_{\bar{H}} + 1/2)k|y|}$, respectively. In general, the GUT states such as the XY and triplet-Higgs states are naturally heavier than the MSSM states, because in the 4D picture they are composite states of G and thus couple more strongly to the supersymmetry breaking than the elementary states. This is strongly related to the fact that we obtain the MSSM prediction for gauge coupling unification, which arises because the GUT states are composite particles so that their contribution to the gauge coupling evolution shuts off above the compositeness scale of order TeV.

So far we have only considered the masses generated at tree level in 5D. The MSSM squarks and sleptons and $A_5^{\rm XY}$ are still massless at this level. They, however, obtain masses at one loop through the standard model gauge interactions. Because of the geometrical separation between supersymmetry breaking and the place where squarks and sleptons are located, the generated squark and slepton masses are finite and calculable in the effective field theory. The calculation of these masses has been carried out in Ref. [10] and the result is given by

$$m_{\tilde{f}}^2 = \frac{1}{2\pi^2} \sum_{a=1,2,3} C_a^{\tilde{f}} \mathcal{I}_a, \tag{41}$$

where $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}, \tilde{e}$ represents the MSSM squarks and sleptons, and the $C_a^{\tilde{f}}$ are the group theoretical factors given by $(C_1^{\tilde{f}}, C_2^{\tilde{f}}, C_3^{\tilde{f}}) = (1/60, 3/4, 4/3), (4/15, 0, 4/3), (1/15, 0, 4/3), (3/20, 3/4, 0)$ and $(3/5, 0, 0)$ for $\tilde{f} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}$ and $\tilde{e}$, respectively. The functions $\mathcal{I}_a$ are defined in Eq. (21) of [10]. Because these masses are generated through gauge interactions, they are flavor universal and the supersymmetric flavor problem is absent. Another important point is that we have to use three different gaugino mass parameters $M_{\lambda,a}$ in the functions $\mathcal{I}_a$ because of the non-universal gaugino masses. This has interesting consequences on the phenomenology of the model as discussed in the next section.

The mass of $A_5^{\rm XY}$ is similarly generated at one loop through gauge interactions. It receives contributions from loops of the 321 gauginos as well as those of the 321 gauge bosons, with the former cutting off the quadratically divergent contribution from the latter. The remaining logarithmic divergence is then made finite at the scale $\pi k'$ by the loops of the KK towers. This is because in the 4D picture $A_5^{\rm XY}$ is the pseudo-Goldstone boson associated with the breaking of the global symmetry of G, $SU(5) \rightarrow 321$, which is encoded as the boundary condition breaking *at the TeV brane*. This implies that $A_5^{\rm XY}$ can receive a mass only through the explicit breaking of the global $SU(5)$ symmetry of G, which corresponds to the boundary condition breaking *at the Planck brane*. The $A_5^{\rm XY}$ mass is then finite and calculable, since $A_5^{\rm XY}$ (and supersymmetry breaking) is localized to the TeV brane so that the loops generating the $A_5^{\rm XY}$ mass must probe both the Planck and the TeV branes. We thus find that the mass of $A_5^{\rm XY}$ is given by

$$m_{A_5^{\rm XY}}^2 \simeq \frac{g^2 C}{\pi^4} m_{\lambda'^{\rm XY}}^2 \ln \frac{\pi k'}{m_{\lambda'^{\rm XY}}}, \tag{42}$$

where g represents a 4D gauge coupling and C the group theoretical factor. This expression is valid as long as $m_{\lambda'^{\rm XY}} \gtrsim m_{\lambda_a^{321}}$, which is expected to be the case. Since $A_5^{\rm XY}$ is charged under $SU(3)_C$, we should take $g = g_3$ and $C = 4/3$ in the above estimate.

To summarize, we have seen that supersymmetry breaking at the TeV brane naturally gives all the masses necessary to make the model viable, with the sizes of the masses depending on the states. It is useful to classify the fields into several categories to figure out the relative sizes of

	(A) "elementary"	(B) "composite"
(I) tree	λ^{321}, $\{H_D, \bar{H}_D$ for $c \simeq 1/2\}$	$\lambda'^{\rm XY}, \sigma^{\rm XY}, H_T^c, \bar{H}_T^c$
(II) loop	$\tilde{f}$, $\{H_D, \bar{H}_D$ for $c \gg 1/2\}$	$A_5^{\rm XY}$

Table 1: The classification of the fields obtaining masses from supersymmetry breaking; see the text.

their masses. We can first divide the fields into two categories, the fields regarded respectively as (A) elementary and (B) composite fields in the 4D picture. The former consists of the MSSM states, $\lambda^{321}, H_D, \bar{H}_D$ and $\tilde{f}$, while the latter consists of the GUT states, $\lambda'^{\rm XY}, \sigma^{\rm XY}, A_5^{\rm XY}, H_T^c$ and $\bar{H}_T^c$. In the 5D picture class (B) corresponds to the states which are localized to the TeV brane, while (A) corresponds to the states which are not. Since supersymmetry breaking is caused by the dynamics of G (localized to the TeV brane in the 5D picture) the states in (B) generically receive larger masses than those in (A). We could also divide the fields into the classes which receive masses at (I) tree and (II) loop levels in 5D. The class (I) contains λ^{321}, $\lambda'^{\rm XY}$, $\sigma^{\rm XY}$, H_T^c and $\bar{H}_T^c$ (and $H_D, \bar{H}_D$ for $c \simeq 1/2$), while the class (II) contains $\tilde{f}$ and $A_5^{\rm XY}$ (and $H_D, \bar{H}_D$ for $c \gg 1/2$). The masses of the fields in (II) are naturally suppressed compared with those in (I) by a loop factor (but the logarithm between the masses and the KK scale could make this hierarchy small, especially when supersymmetry breaking is weak). The fields belonging to the four classes (A-I), (A-II), (B-I) and (B-II) are depicted explicitly in Table 1, and the general trend for the spectrum in the gauge sector is depicted in Fig. 1b. Assuming that coefficients of all the operators scale according to naive dimensional analysis, the ratios of the masses for fields in each category are estimated roughly as $m_{\rm A-I} : m_{\rm A-II} : m_{\rm B-I} : m_{\rm B-II} \approx 1 : g_B^2 k/16\pi^2 : \pi kR : (g^2 C/\pi^4)^{1/2} \pi kR$. We should, however, emphasize that the operators giving the masses for these states have coefficients which are free parameters (see e.g. Eqs. (19, 21, 23, 25)). In particular, the operators giving masses for $\lambda'^{\rm XY}$ and $\sigma^{\rm XY}$ involve the y derivative so that they may have somewhat suppressed coefficients. Therefore, it could well be possible that some of these states, for example $\lambda'^{\rm XY}$, $\sigma^{\rm XY}$ and $A_5^{\rm XY}$, are lighter than the naive estimate given above, improving the prospects for their discovery at future colliders.

4 Phenomenology

For a theory that incorporates something that closely resembles MSSM gauge coupling unification, the spectrum of MSSM particles in the present model can be quite unusual. Because the gaugino masses originate from supersymmetry breaking terms on the $SU(5)$-violating TeV brane, there is no guarantee that the gluino will be much heavier than the wino, which will in turn

be heavier than the bino. Moreover, the squark and slepton masses are generated radiatively through gaugino loops, as determined by Eq. (41), so the scalar spectrum inherits whatever unusual features distinguish the gaugino spectrum.

To obtain the physical masses of the gauginos and scalars, the values given by Eqs. (20, 41), which are the running masses at the KK scale $m_{\rm KK} \simeq \pi k'$, must be run down in energy. For the scalar masses-squared, the D-term contributions $\Delta = (T_3 - Q\sin^2\theta_W)\cos 2\beta\, m_Z^2$ must also be included. The smaller the size of supersymmetry breaking on the TeV brane, the larger the energy interval between $m_{\rm KK}$ and the superpartner masses, and the larger the running effect will be. For very strong supersymmetry breaking, the squark and slepton masses will be considerably smaller than the gaugino masses, while for weaker supersymmetry breaking masses, the scalar and gaugino masses can be comparable. This is evident in the plots of [10], where with $x \equiv M_\lambda/k$, the ratio of the TeV-brane gaugino mass to the curvature scale, ranging from 10 to 0.01, the ratio of the bino mass to the right-handed selectron mass falls from roughly a factor of eight to less than a factor of two.

One interesting consequence of the non-universality of the TeV-brane gaugino masses is that there are many possibilities for which particle is the next-to-lightest supersymmetric particle (NLSP) (the NLSP decays promptly into the LSP gravitino, whose mass is $\sim k'^2/M_{\rm Pl} \sim 0.01 - 0.1$ eV). If the TeV-brane gaugino masses are universal as in [10], the NLSP will be the right-handed stau. More generally, this is the result when the dominant contributions to the scalar masses are from gluino and wino loops – in the notation of Eq. (41), $\mathcal{I}_3,\ \mathcal{I}_2 > \mathcal{I}_1$. If instead $\mathcal{I}_3,\ \mathcal{I}_1 > \mathcal{I}_2$, it is possible for the bino-loop contribution to $m_{\tilde{e}}^2$ to be larger than the wino-loop contribution to $m_{\tilde{l}}^2$, giving a sneutrino NLSP. A right-handed bottom squark NLSP is possible if $\mathcal{I}_2,\ \mathcal{I}_1 > \mathcal{I}_3$, and even the left-handed stop could technically be the NLSP, although it requires $\mathcal{I}_1 \gg\ \mathcal{I}_2,\ \mathcal{I}_3$. In fact, since all of the scalars get contributions from bino loops, one can even have scenarios with either wino or gluino NLSP's. Some of these cases are likely to be more natural than others from the perspective of electroweak symmetry breaking; here we have simply enumerated some of the various possibilities.

In the limit that the squarks and sleptons are heavy enough that D-term splittings are not very important, the three quantities $\mathcal{I}_1$, $\mathcal{I}_2$, and $\mathcal{I}_3$ determine the five masses for $\tilde{q}$, $\tilde{u}$, $\tilde{d}$, $\tilde{l}$, and $\tilde{e}$. If the gluino contribution dominates the masses of $\tilde{q}$, $\tilde{u}$, and $\tilde{d}$, they will be quite degenerate, and only by resolving this degeneracy can the sum rules

$$m_{\tilde{q}}^2 = m_{\tilde{d}}^2 + m_{\tilde{l}}^2 - \frac{1}{3}m_{\tilde{e}}^2, \tag{43}$$

$$m_{\tilde{u}}^2 = m_{\tilde{d}}^2 + \frac{1}{3}m_{\tilde{e}}^2, \tag{44}$$

be tested. On the other hand, if $\mathcal{I}_1$, $\mathcal{I}_2$, and $\mathcal{I}_3$ are all comparable, there will not be a strong hierarchy among the scalar masses, and the above relations will be more easily tested. Such a

scenario requires the brane mass for the gluino to be somewhat smaller than that for the wino, for instance. In reality, it is quite possible that D-term contributions will be important, in which case the above sum rules will be modified. In that case, however, there are four parameters (now including $\tan\beta$) that fix seven scalar masses, so the more general point still remains — the model predicts relations among the scalar masses, and these will be most easily tested if the scalar masses are all comparable in size.[7]

A very predictive spectrum arises in the limit of very strong supersymmetry breaking on the TeV brane, in which case the entire mass spectrum of the theory is essentially determined by the single free parameter k'. This spectrum for the MSSM gauginos and scalars is identical to the one described in [10] for $x \gg 1$. A distinctive feature is that the MSSM gauginos combine with the conjugate gauginos from Σ^{321} to form pseudo-Dirac states in this limit. An important difference compared to [10] is that here the XY gauginos do not become massless in the limit of infinitely strong supersymmetry breaking. Instead, $\lambda^{\rm XY}$ and $\lambda'^{\rm XY}$ appear in Dirac states, the lightest of which are nearly degenerate with the lightest KK excitations of the standard-model gauge bosons, $m \simeq (3\pi/4)k'$. The lightest $\sigma^{\rm XY}$ mode also has this mass, as can be verified using Eq. (24). The mass of the lightest $A_5^{\rm XY}$ mode is of order k'/π.

As discussed in section 3, the lightest of the GUT particles (LGP) is likely to be $A_5^{\rm XY}$ because its mass is loop suppressed. The LGP, which is generally colored, will be stable at least for collider purposes. This is because the LGP is localized to the TeV brane, so that its decay to quarks is highly suppressed (there is no bulk mode that can carry a color charge from the TeV brane to the Planck brane at a sufficiently large rate).[8] Since the LGP is colored, it hadronizes after production. These exotic hadrons are charged or neutral, depending on whether the LGP picks up an up or down quark or antiquark, but the mass differences among them are small enough that they are both stable for collider purposes. The charged ones would thus be detectable at collider experiments through highly ionizing tracks [2]. In the case that $A_5^{\rm XY}$ is the LGP, the exotic hadrons are four fermionic mesons: $\tilde{T}^0$, $\tilde{T}^-$, $\tilde{T}'^-$ and $\tilde{T}^{--}$ (and their anti-particles). We can estimate the reach of the LHC to be roughly 2 TeV in the masses of these states [22].

Finally, we make a few remarks about the Higgs sector in the model. If the Higgs multiplets are strongly localized to the Planck brane, their soft masses-squared are generated radiatively just as for the other MSSM scalars. At one loop, wino and bino loops give a positive contribution equal to the one-loop contribution to $m_{\tilde{l}}^2$. A two-loop negative contribution to m_H^2 coming from the top Yukawa interaction will overcome this positive contribution provided the gluino mass is not too small. In this case of brane-localized Higgs fields, some mechanism for generating μ must be introduced. One possibility is to generate μ by "shining" it from the TeV brane, as described

[7]For the third generation, the relations described here will be modified by Yukawa effects.
[8]The GUT parity of [2] can be badly broken on the TeV brane in the present model.

in [2]; μB can then be generated radiatively through gaugino loops unless the supersymmetry breaking on the TeV brane is so strong that the gauginos are essentially Dirac in nature. If the Higgs fields are delocalized, then μ, μB, m_H^2, and $m_{\bar{H}}^2$ all arise at tree level. For these parameters to be generated with the appropriate size for natural electroweak symmetry breaking, the Higgs hypermultiplet mass parameters should be close to their sizes in the conformal limit: $c_H \sim c_{\bar{H}} \simeq 0.5 - 0.6$ (see Eqs. (34 – 36)).

5 Conclusions

We have studied a model of warped supersymmetric unification in which neither the Planck brane nor the TeV brane respects the $SU(5)$ unified symmetry. Before supersymmetry breaking, the massless spectrum contains fermions and scalars with the quantum numbers of XY gauge fields, in addition to the particle content of the MSSM. In the 4D description, these states make up the pseudo-Goldstone supermultiplets associated with the spontaneous breaking of the approximate $SU(5)$ global symmetry. We have shown that these states can acquire masses from supersymmetry breaking terms on the TeV brane, and that they leave intact the unification prediction for the low-energy gauge couplings. Whether they will be accessible to future colliders such as the LHC depends on the sizes of coefficients of operators that produce their masses (e.g. η and ρ in Eqs. (21, 23)). The lightest of these states are expected to be the scalars $A_5^{\rm XY}$, which have loop-suppressed masses.

The model we have presented leads to a spectrum of MSSM gauginos and scalars that can be quite unusual. This stems from the fact that these particles acquire mass (either at tree level or radiatively) from supersymmetry breaking terms on the TeV brane, where the unified symmetry is not realized. Consequently, while the model is valid as an effective theory up to very high scales and incorporates gauge coupling unification, aspects of the spectrum that are essentially fixed in more conventional supersymmetric unification — for example, the ordering of the gaugino masses — are here allowed a range of possibilities.

Acknowledgment

The work of Y.N. and B.T. was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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